$$\frac{1}{\sqrt{2}}\int_0^1 x^n\,dx \leq U_n \leq \int_0^1 x^n\,dx$$

$$\frac{1}{\sqrt{2}}\left[\frac{1}{n+1}x^{n+1}\right]_0^1 \leq U_n \leq \left[\frac{1}{n+1}x^{n+1}\right]_0^1$$

$$\frac{1}{\sqrt{2}}\cdot\frac{1}{n+1} \leq U_n \leq \frac{1}{n+1}$$

Donc $$\boxed{\frac{1}{\sqrt{2}(n+1)} \leq U_n \leq \frac{1}{n+1}}$$

Montrer que : $\frac{1}{(n+1)\sqrt{2}} \leq U_n \leq \frac{1}{n+1}$. En déduire la limite de la suite (U_n)

On $\lim\limits_{n\to+\infty} \frac{1}{\sqrt{2}(n+1)} = 0$

et

$$\lim_{n\to+\infty} \frac{1}{n+1} = 0$$

Donc $$\boxed{\lim_{n\to+\infty} U_n = 0}$$